## Lecture 4. Classification of systems

**The purpose of the lecture:** to discuss an introduction to methods of classifying systems, large and complex systems.

**Lecture plan:**
Introduction
1 System classification
Conclusion

**Keywords:** software, classification method, self-organizing, computer, externally controlled, logical, disk, firm, internally controlled, large, modeling, set of states, dimension, large system, system, parallel structures, complex system, decision making, complex, Configurator , assessment of adequacy, external, internal, internal complexity, sets, external complexity, logical description, functioning algorithm, functional description, self-organization, system of linear algebraic equations, matrix, solvability, inverse, Gauss method, dynamical system, Cauchy problem, computation errors, Internet, connectivity, telecommunication network, types of connections, fraction, object, self-similarity, branch, visualization, connected, measure, probability, function, difference, stable system, controllers, formal system, operations, test set, logical operator, PROLOGUE, veronym, Outcome, communication, analysis, optimization, common sense, statistics, game theory, fuzzy stiffness, fuzzy logic, heuristic programming, isomorphism.

**Contents of the lecture:**

### Introduction

Systems can be classified according to different criteria. Carrying it out toughly is impossible, it depends on the goal and resources.

### 1 System classification

Here are the main classification methods (other criteria for classifying systems are also possible).

1. The relation of the system to the environment:
   - open (there is an exchange of resources with the environment);
   - closed (there is no exchange of resources with the environment).
2. By the origin of the system (elements, connections, subsystems):
   - artificial (tools, mechanisms, machines, automata, robots, etc.);
   - natural (living, non-living, ecological, social, etc.);
   - virtual (imaginary and, although not really existing, but functioning in the same way as if they existed);
   - mixed (economic, biotechnical, organizational, etc.).
3. According to the description of system variables:
   - with qualitative variables (having only a meaningful description);

- with quantitative variables (having discretely or continuously quantitatively described variables);
- mixed (quantitative and qualitative) description.

4. By the type of description of the law (laws) of the functioning of the system:

- type "Black box" (the law of the system's functioning is completely unknown; only input and output messages are known);
- not parameterized (the law is not described; we describe using at least unknown parameters; only some a priori properties of the law are known);
- parameterized (the law is known up to parameters and it is possible to refer it to a certain class of dependencies);
- type "White (transparent) box" (the law is fully known).

5. By the way the system (in the system) is managed:

- externally controlled systems (without feedback, regulated, controlled structurally, informationally or functionally);
- controlled from within (self-controlled or self-regulating - programmatically controlled, automatically regulated, adaptable - adaptable with the help of controlled changes in states, and self-organizing - changing their structure in time and space most optimally, ordering their structure under the influence of internal and external factors);
- with combined control (automatic, semi-automatic, automated, organizational).

*Example.* Let's consider the ecological system "Lake". This is an open system of natural origin, the variables of which can be described in a mixed way (quantitatively and qualitatively, in particular, the temperature of the reservoir is a quantitatively described characteristic), the structure of the lake inhabitants can be described both qualitatively and quantitatively, and the beauty of the lake can be described qualitatively. By the type of description of the system functioning law, this system can be classified as non-parametrized in general, although it is possible to distinguish subsystems of various types, in particular, different descriptions of the subsystems "Algae", "Fish", "Flowing stream", "Outflowing stream", "Bottom", "Coast", etc. The "Computer" system is open, artificial, mixed description, parameterized, controlled from the outside (programmatically). The "Logical Disk" system is an open, virtual, quantitative description, of the "White Box" type (we do not include the contents of the disk in this system!), Mixed management. The "Firm" system is open, of mixed origin (organizational) and descriptions, managed from within (an adaptable system, in particular).

A system is called ***large*** if its study or modeling is difficult due to its large dimension, i.e. the set of states of the system S has a large dimension. What dimension should be considered large? We can judge about this only for a specific problem (system), a specific goal of the problem under study and specific resources.

A large system is reduced to a system of lower dimension using more powerful computational tools (or resources) or dividing the problem into a number of problems of lower dimension (if possible).

*Example*. This is especially important in the development of large computing systems, for example, in the development of computers with a parallel architecture or algorithms with a parallel data structure and with their parallel processing.

In almost all textbooks one can find the phrases "difficult task", "difficult problem", "complex system", etc. Intuitively, as a rule, these concepts are understood as some special behavior of a system or process that makes it impossible (insurmountable complexity) or especially difficult (surmountable complexity) to describe, research, predict or evaluate the behavior and development of the system.

The definitions of complexity are different.

A system is called ***complex*** if it lacks resources (mainly information) for an effective description (of states, laws of operation) and control of the system - definition, description of control parameters or for making decisions in such systems (such systems should always have a subsystem of solutions).

Sometimes a system is considered complex, for which its trajectory, essence cannot be revealed by its three types of description, and therefore an additional integral description (an integral model of behavior, or configurator) is still needed - morphological-functional-infological.

*Example*. Complex systems are, for example, chemical reactions, if studied at the molecular level; a cell of biological formation taken at the metabolic level; the human brain, if examined in terms of the intellectual actions performed by a person; the economy considered at the macro level (i.e. macroeconomics); human society - at the political, religious and cultural level; Computers (especially the fifth generation) as a means of acquiring knowledge; language - in many aspects of its consideration.

In complex systems, the result of functioning cannot be specified in advance, even with some probabilistic assessment of adequacy. The reasons for this uncertainty are both external and internal, both in the structure and in the description of functioning and evolution. The complexity of these systems is due to their complex behavior. The complexity of the system depends on the accepted level of description or study of the system - macroscopic or microscopic. The complexity of a system can be determined not only by a large number of subsystems and a complex structure, but also by the complexity of behavior.

The complexity of the system can be external and internal.

***Internal complexity*** is determined by the complexity of the set of internal states, potentially estimated by the manifestations of the system and the complexity of control in the system.

***External complexity*** is determined by the complexity of the relationship with the environment, the complexity of managing the system, potentially estimated by the feedback of the system and the environment.

Complex systems come in different types of complexity:

- ✓ *structural or organizational (not enough resources to build, describe, manage the structure);*
- ✓ *dynamic or temporal (there are not enough resources to describe the dynamics of the system's behavior and control its trajectory);*

- ✓ *informational or information-logical, infological (there are not enough resources for information, information-logical description of the system);*
- ✓ *computational or implementation, research (there are not enough resources for effective forecasting, calculations of system parameters, or their implementation is difficult due to lack of resources);*
- ✓ *algorithmic or constructive (there are not enough resources to describe the algorithm for the functioning or control of the system, for a functional description of the system);*
- ✓ *development or evolution, self-organization (there are not enough resources for sustainable development, self-organization).*

The more complex the system under consideration, the more varied and more complex internal information processes have to be updated in order to achieve the goal of the system, i.e. the system was functioning or evolving.

*Example*. The behavior of a number of different real systems (for example, interconnected conductors with resistances $x_1, x_2, ..., x_n$ or chemical compounds with concentrations $x_1, x_2, ..., x_n$, participating in the reaction of chemical reagents) is described by a system of linear algebraic equations, written in matrix form: X = AX + B where

$$X = \begin{Vmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{Vmatrix}, \quad A = \begin{Vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{Vmatrix}, \quad B = \begin{Vmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{Vmatrix}.$$

Filling in the matrix A (its structure) will reflect the complexity of the described system. If, for example, the matrix A is an upper triangular matrix (the element located at the intersection of the i-th row and the j-th column is always 0 for $i > j$), then regardless of n (the dimension of the system) it is easily investigated for solvability. To do this, it is sufficient to reverse the Gauss method. If the matrix A is of general form (it is neither symmetric, nor banded, nor sparse, etc.), then the system is more difficult to investigate (since in this case it is necessary to perform a more complex computationally and dynamically procedure for the forward run of the Gauss method). Consequently, the system will have structural complexity (which may already entail computational complexity, for example, when finding a solution). If the number n is large enough, then the unsolvability of the problem of storing the upper triangular matrix A in the computer's RAM can cause the computational and dynamic complexity of the original problem. An attempt to use this data by reading from the disk will lead to a manifold increase in the computation time (it will increase the dynamic complexity - the factors of working with the disk will be added).

*Example.* Let there be a dynamical system whose behavior is described by a Cauchy problem of the form

$$y'(t) = ky(t),\ y(0) = a.$$

This task has a solution:

$$y(t) = ae^{-kt}$$

Hence, it can be seen that y(t) at k = 10 changes an order of magnitude faster than y(t) at k = 1, and the dynamics of the system will be more difficult to track: a more accurate prediction for $t \to 0$ and small k is associated with additional costs for calculations. Consequently, algorithmically, informationally, dynamically and structurally, a "not very complex system" (for a, $k \neq 0$) can become computationally and possibly evolutionarily complex (for $t \to 0$), and for large t ($t \to \infty$) - and unpredictable. For example, for large t, the values of the accumulated calculation errors of the solution may overlap the values of the solution itself. If in this case zero initial data $a \neq 0$ are specified, then the system can cease to be, for example, informationally simple, especially if it is difficult to determine a priori.

*Example*. Simplification of technical means of working in networks, for example, scientific advances that allow you to connect a computer directly to a network, "to an electrical outlet", is observed along with the complication of the networks themselves, for example, with an increase in the number of subscribers and information flows on the Internet. Along with the complication of the Internet itself, the means of access to it are simplified (for the user!), And its computing capabilities increase.

The structural complexity of the system affects the dynamic, computational complexity. Changes in dynamic complexity can lead to changes in structural complexity, although this is not required. A complex system can also be a system that is not a large system; in this case, the connectivity (strength of connectivity) of elements and subsystems of the system can become essential (see the above example with the matrix of a system of linear algebraic equations).

The complexity of a system is determined by goals and resources (a set of tasks that it is designed to solve).

*Example.* The complexity of a telecommunications network is determined by:

1. *required data transfer rate;*
2. *protocols, communications and types of communications (for example, a conference call requires a voice teleconference);*
3. *the need for video support.*

The very concept of the complexity of a system is not something universal, unchanging, and can change dynamically, from state to state. At the same time, weak links and relationships between subsystems can increase the complexity of the system.

*Example*. Consider the procedure for dividing the unit segment [0; 1], followed by discarding the middle of three segments and completing an equilateral triangle on the discarded segment (Fig. 4.1); we will repeat this procedure each time again for each of the segments remaining after discarding. This process is structurally simple, but dynamically complex, moreover, a dynamically interesting and difficult to trace picture of a system is formed, which is becoming "more and more, more and more complex." Such structures are called fractals, or fractal structures (fractal - from fraction - "fraction" and fracture - "break", that is, a broken object with a fractional dimension). Its distinguishing feature is self-similarity, i.e. an arbitrarily small part of a fractal is structurally similar to the whole, as a branch is to a tree.

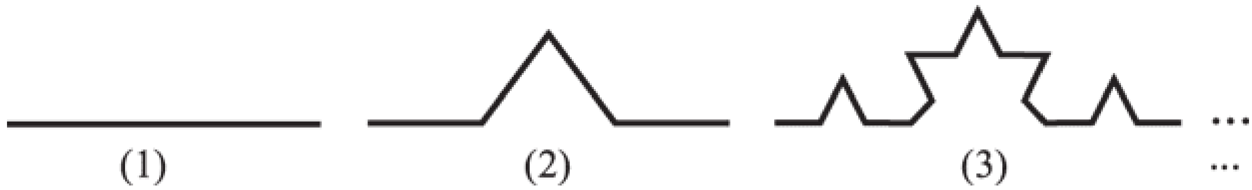


Figure 4.1. Fractal object (Koch curve)

By reducing the complexity of the system, it is often possible to increase its information content and investigability.

*Example.* The choice of a rational projection of a spatial object (i.e., a more optimal visualization of the connections and relationships of its parts) makes the drawing more informative. Using a microscope as an experimental device, you can see some properties of an object invisible to the naked eye.

A system is called ***connected*** if any two subsystems exchange a resource, i.e. there are some resource-oriented relationships, connections between them.

When defining a measure of the complexity of a system, it is important to single out the invariant properties of systems or information invariants and introduce a measure of the complexity of systems based on their descriptions.

Here is a mathematical apparatus that allows you to formalize the concept of complexity, although we note that the concept of complexity is "complex".

A measure below will mean some continuous real nonnegative function defined on a set of events (systems, sets) and which is additive, i.e. the measure of the final union of events (systems, sets) is equal to the sum of the measures of each event.

How to determine the degree of complexity for systems of various structures? The answer to this equally difficult question cannot be unambiguous and even quite definite.

Complexity is associated with the measure $\mu(S)$ - a measure of complexity or a numerical non-negative function (criterion, scale) given on a certain set of elements and subsystems of the system S.

There are various ways to determine the measure of the complexity of systems. The complexity of the structure of a system can be determined by topological entropy - the complexity of the configuration of the structure (system):

$$S = k \ln W,$$

where $k = 1{,}38\times10^{-16}$ (erg/deg) is the Boltzmann constant, W is the probability of the state of the system. In the case of different probabilities of states, this formula will have the form (below we will return to a detailed discussion of this formula and its various modifications):

$$S = -k \sum_{i=1}^{n} p_i \ln p_i$$

*Example*. Let's define the complexity of the hierarchical system as the number of levels of the hierarchy. The increase in complexity in this case requires more resources to achieve the goal. Let us define the complexity of a linear structure as the number of subsystems in the system. Let us define the complexity of the network structure as the maximum of the complexity of all linear structures corresponding to various strategies for achieving the goal (paths leading from the initial subsystem to the final one). The complexity of a system with a matrix structure can be determined by the number of subsystems in the system. The complication of a certain subsystem of the system will lead to the complication of the entire system in the case of a linear structure, and, possibly, in the case of hierarchical, network and matrix structures.

*Example*. For polyatomic molecules, the number of internuclear distances (it determines the configuration of the molecule) can be considered an estimate of the complexity of the topology (geometric complexity) of the molecule. This estimate is known from chemistry and mathematics: 3N-6, where N is the number of atoms in a molecule. For solid solutions, W can be considered equal to the number of permutations of atoms of different types in the given positions of the structure; for a pure crystal W = 1, for a mixed one - W> 1. For a pure crystal, the complexity of the structure is S = 0, and for a mixed one – S > 0, which is to be expected.

*Example*. In ecological and economic systems, the complexity of a system can often be understood as the complexity of the evolution of the system, in particular, a measure of complexity - as a function of changes occurring in the system as a result of contact with the environment, and this measure can be determined by the complexity of the interaction between the system (organism, organization) and environment, its controllability. The evolutionary complexity of an evolving system can be defined as the difference between internal complexity and external complexity (the complexity of complete system management). Decisions in these systems should be made (for the stability of the systems) in such a way that the evolutionary complexity is equal to zero, i.e. so that internal and external difficulties coincide. The smaller this difference, the more stable the system, for example, the more balanced the intramarket relations and the government influences governing them, the more stable the market and market relations.

*Example*. In mathematical, formal systems, the complexity of a system can be understood as algorithmicability, computability of the operator of the system S, in particular, as the number of operations and operands required to obtain the correct result for any admissible input set. The complexity of the algorithm can be determined by the number of operations performed by the instructions of the algorithm for the "worst" (the longest on the way to achieving the goal) test data set.

*Example*. The complexity of the software package L can be defined as logical complexity and measured as $L = L_1/L_2 + L_3 + L_4 + L_5$, where $L_1$ is the total number of all logical operators, $L_2$ is the total number of all executable operators, $L_3$ is the indicator of the complexity of all loops (is determined using the number of loops and their nesting), $L_4$ is an indicator of the complexity of loops (determined by the number of conditional operators at each nesting level), $L_5$ is determined by the number of branches in all conditional operators.

*Example*. Similar to the example given in the book by J. Casti, consider the tragedy of W. Shakespeare "Romeo and Juliet". Let's select and describe 3 sets: A - play, acts, scenes, mizzen-scenes; B - characters; C - comments, play, plot, phenomenon, remarks. Let's define the hierarchical levels and elements of these aggregates.

A:

level N + 2 - Play;

level N + 1 - Acts {a1, a2, a3, a4, a5};

level N - Scenes {s1, s2, ..., sq};

level N-1 - Mizzen-scenes {m1, m2, ..., m26}.

B:

level N - Characters $\{c_1, c_2, ..., c_{25}\}$ = {Romeo, Juliet, ...}.

C:

level N + 3 - Prologue (addressed directly to the viewer and lies outside the actions that unfold in the play);

level N + 2 - Play;

level N + 1 - Storylines $\{p_1, p_2, p_3, p_4\}$ = {The feud of the Capulet and Montague families in Verona, The love of Juliet and Romeo and their wedding, The murder of Tybalt and the feud of families requires revenge, Romeo is forced into hiding, The marriage of Paris to Juliet , Tragic outcome};

level N - Phenomena $\{u_1, u_2, ..., u_8\}$ = {Love of Romeo and Juliet, Relationship between the family of Capulet and Montague, Wedding of Romeo and Juliet, Fight of Romeo and Tybalt, Romeo is forced into hiding, Marriage of Paris, Juliet's decision, Doom lovers};

level N-1 - Remarks $\{r_1, r_2, ..., r_{104}\}$ = {104 replicas in the play, which are defined as words addressed to the viewer, the actor and developing a plot unknown to the viewer}.

The relationships between these aggregates at different levels of the hierarchy are determined from these aggregates. For example, if Y are plots, X are actors, then it is natural to define the relationship l between X, Y as follows: an actor from the set X of level N + 1 participates in the plot Y of level N + 1. Then the coherence of the structure of the tragedy can be depicted in the following diagram (Fig.4.2):

In this complex K(Y, X), all three plots become separate components only at the level of connectivity q = 8. This means that the storylines can only be different for the audience following the 9 characters. Similarly, for q = 6, there are only 2 components $\{p_1, p_2\}$, $\{p_3\}$. Therefore, if viewers can only track 7 characters, then they see a play, as it were, consisting of two plots, where $p_1$, $p_2$ (the world of lovers and the enmity of families) are combined. The complex K(Y, X) for q = 5 has 3 components. Consequently, viewers who have seen only 6 scenes perceive 3 scenes that are not related to each other. Plots $p_1$ and $p_2$ are combined at q = 4, and therefore viewers can see these two plots as one if they are watching only 5 scenes. All 3 plots merge when viewers follow only 3 scenes. In the K(Y, X) complex, the $u_8$

phenomenon dominates in the structure at q = 35, $u_3$ at q = 26, and $u_6$ at q = 10. Therefore, $u_8$ is most likely to be understood by those viewers who have listened to 36 cues, although understanding $u_3$ requires 27 cues, and understanding $u_6$ only 11 cues. Thus, the analysis performed provides an understanding of the complexity of the system.

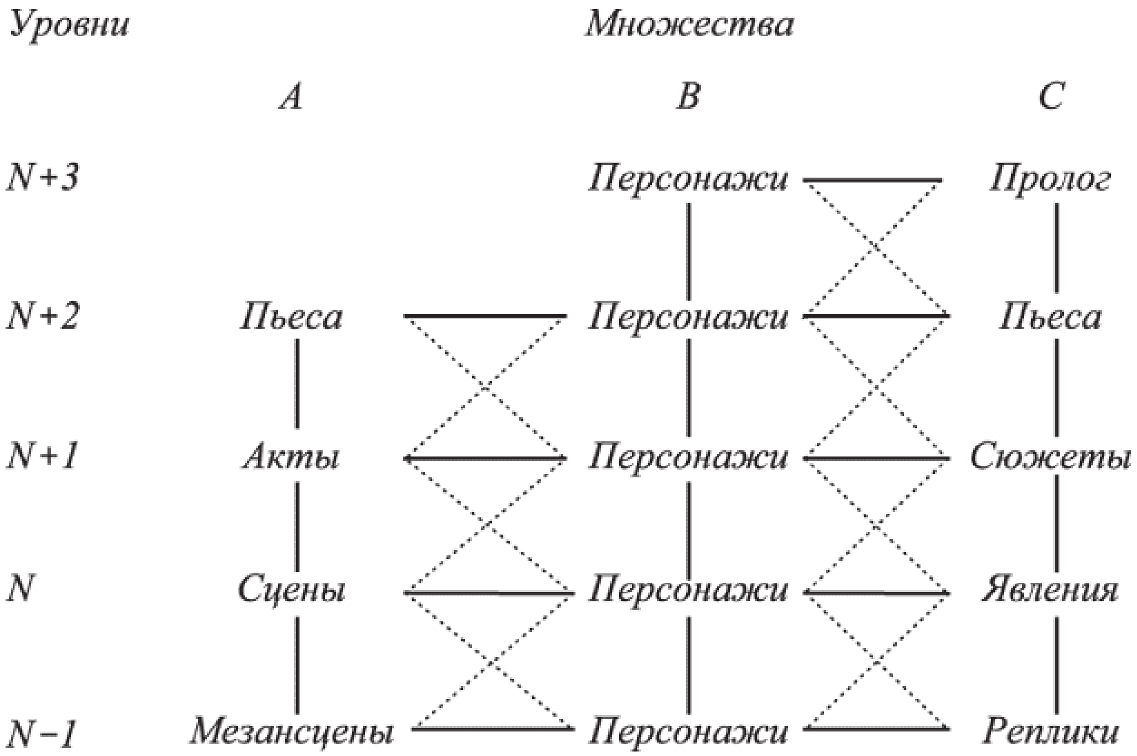

Figure 4.2. Scheme of structural connections of the play

Recently, the so-called "hard" and "soft" systems began to be distinguished, mainly according to the criteria used for consideration.

**Conclusion**

The study of "rigid" systems is usually based on the categories: "design", "optimization", "implementation", "goal function" and others. For "soft" systems, the following categories are used more often: "possibility", "desirability", "adaptability", "common sense", "rationality" and others. The methods are also different: for "rigid" systems - optimization methods, probability theory and mathematical statistics, game theory and others; for "soft" systems - multicriteria optimization and decision making (often under uncertainty), the Delphi method, catastrophe theory, fuzzy sets and fuzzy logic, heuristic programming, etc. For the "transfer" of knowledge, system invariants and system isomorphism are widely used. It is important with such a transfer not to violate the system's emergence property.

**Control questions**

See the manual on the organization of students' independent work.